1.

$$\frac{2x^2 + 5x - 10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}$$

(a) Find the values of the constants A, B and C.

(4)

(b) Hence, or otherwise, expand $\frac{2x^2 + 5x - 10}{(x-1)(x+2)}$ in ascending powers of x, as far as the term in x^2. Give each coefficient as a simplified fraction.

(7)
(Total 11 marks)

2. (a) Find the binomial expansion of

$$\sqrt{(1-8x)}, \quad |x| < \frac{1}{8},$$

in ascending powers of x up to and including the term in x^3, simplifying each term.

(4)

(b) Show that, when $x = \frac{1}{100}$, the exact value of $\sqrt{(1 - 8x)}$ is $\frac{\sqrt{23}}{5}$.

(2)

(c) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (a) and hence obtain an approximation to $\sqrt{23}$. Give your answer to 5 decimal places.

(3)
(Total 9 marks)

3. $$\text{f}(x) = \frac{1}{\sqrt{(4+x)}}, \qquad |x| < 4$$

Find the binomial expansion of f (x) in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(Total 6 marks)

4. $$\text{f}(x) = \frac{27x^2 + 32x + 16}{(3x+2)^2(1-x)}, |x| < \frac{2}{3}$$

Given that f(x) can be expressed in the form

$$\text{f}(x) = \frac{A}{(3x+2)} + \frac{B}{(3x+2)^2} + \frac{C}{(1-x)},$$

(a) find the values of B and C and show that $A = 0$.

(4)

(b) Hence, or otherwise, find the series expansion of f(x), in ascending powers of x, up to and including the term in x^2. Simplify each term.

(6)

(c) Find the percentage error made in using the series expansion in part (b) to estimate the value of f (0.2). Give your answer to 2 significant figures.

(4)

(Total 14 marks)

5. (a) Expand $\frac{1}{\sqrt{(4-3x)}}$, where $|x| < \frac{4}{3}$, in ascending powers of x up to and including the term in x^2. Simplify each term.

(5)

(b) Hence, or otherwise, find the first 3 terms in the expansion of $\dfrac{x+8}{\sqrt{(4-3x)}}$ as a series in ascending powers of x.

(4)
(Total 9 marks)

6. (a) Use the binomial theorem to expand

$$(8-3x)^{\frac{1}{3}}, \ |x| < \frac{8}{3},$$

in ascending powers of x, up to and including the term in x^3, giving each term as a simplified fraction.

(5)

(b) Use your expansion, with a suitable value of x, to obtain an approximation to $\sqrt[3]{(7.7)}$
Give your answers to 7 decimal places.

(2)
(Total 7 marks)

7.

$$f(x) = (3+2x)^{-3}, \quad |x| < \frac{3}{2}.$$

Find the binomial expansion of f(x), in ascending powers of x, as far as the term in x^3.

Give each coefficient as a simplified fraction.

(Total 5 marks)

8.

$$f(x) = (2-5x)^{-2}, \quad |x| < \frac{2}{5}.$$

Find the binomial expansion of f(x), in ascending powers of x, as far as the term in x^3, giving each coefficient as a simplified fraction.

(Total 5 marks)

9. $$f(x) = \frac{3x-1}{(1-2x)^2} \qquad |x| < \tfrac{1}{2}.$$

Given that, for $x \neq \frac{1}{2}$, $\frac{3x-1}{(1-2x)^2} = \frac{A}{(1-2x)} + \frac{B}{(1-2x)^2}$, where A and B are constants,

(a) find the values of A and B.

(3)

(b) Hence, or otherwise, find the series expansion of f(x), in ascending powers of x, up to and including the term in x^3, simplifying each term.

(6)
(Total 9 marks)

10. (a) Find the first four terms in the expansion, in ascending powers of x, of

$$(1-2x)^{-\frac{1}{2}} \qquad |2x| < 1,$$

giving each term in its simplest form.

(4)

(b) Hence write down the first four terms in the expansion, in ascending powers of x, of

$$(100-200x)^{-\frac{1}{2}}, \quad |2x| < 1.$$

(2)
(Total 6 marks)

11.

$$f(x) = \frac{3x^2+16}{(1-3x)(2+x)^2} = \frac{A}{(1-3x)} + \frac{B}{(2+x)} + \frac{C}{(2+x)^2}, \qquad |x| < \tfrac{1}{3}$$

(a) Find the values of A and C and show that $B = 0$.

(4)

(b) Hence, or otherwise, find the series expansion of f(x), in ascending powers of x, up to and including the term in x^3. Simplify each term.

(7)
(Total 11 marks)

12. Use the binomial theorem to expand

$$\sqrt{(4 - 9x)}, \qquad |x| < \frac{4}{9},$$

in ascending powers of x, up to and including the term in x^3, simplifying each term.

(Total 5 marks)

13.
$$f(x) = \frac{1}{\sqrt{(1-x)}} - \sqrt{(1+x)}, \qquad -1 < x < 1.$$

(a) Find the series expansion of f(x), in ascending powers of x, up to and including the term in x^3.

(6)

(b) Hence, or otherwise, prove that the function f has a minimum at the origin.

(4)
(Total 10 marks)

14. Given that

$$\frac{3+5x}{(1+3x)(1-x)} \equiv \frac{A}{1+3x} + \frac{B}{1-x},$$

(a) find the values of the constants A and B.

(3)

(b) Hence, or otherwise, find the series expansion in ascending powers of x, up to and including the term in x^2, of

$$\frac{3+5x}{(1+3x)(1-x)}.$$

(5)

(c) State, with a reason, whether your series expansion in part (b) is valid for $x = \frac{1}{2}$.

(2)
(Total 10 marks)

15. $$f(x) = \frac{25}{(3+2x)^2(1-x)}, \quad |x| < 1.$$

(a) Express f(x) as a sum of partial fractions.

(4)

(b) Hence find $\int f(x)\,dx$.

(5)

(c) Find the series expansion of f(x) in ascending powers of x up to and including the term in x^2. Give each coefficient as a simplified fraction.

(7)
(Total 16 marks)

16. (a) Expand $(1 + 3x)^{-2}$, $|x| < \frac{1}{3}$, in ascending powers of x up to and including the term in x^3, simplifying each term.

(4)

(b) Hence, or otherwise, find the first three terms in the expansion of $\frac{x+4}{(1+3x)^2}$ as a series in ascending powers of x.

(4)
(Total 8 marks)

17. The binomial expansion of $(1+12x)^{\frac{3}{4}}$ in ascending powers of x up to and including the term in x^3 is

$$1 + 9x + px^2 + qx^3, \; |12x| < 1.$$

(a) Find the value of p and the value of q.

(4)

(b) Use this expansion with your values of p and q together with an appropriate value of x to obtain an estimate of $(1.6)^{\frac{3}{4}}$.

(2)

(c) Obtain $(1.6)^{\frac{3}{4}}$ from your calculator and hence make a comment on the accuracy of the estimate you obtained in part (*b*).

(2)
(Total 8 marks)

18. $\mathrm{f}(x) = \dfrac{1+14x}{(1-x)(1+2x)}, \qquad |x| < \dfrac{1}{2}.$

(a) Express f(x) in partial fractions.

(3)

(b) Hence find the exact value of $\displaystyle\int_{\frac{1}{6}}^{\frac{1}{3}} \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in the form ln p, where p is rational.

(5)

(c) Use the binomial theorem to expand f(x) in ascending powers of x, up to and including the term in x^3, simplifying each term.

(5)
(Total 13 marks)

1. (a) $A = 2$ B1

$2x^2 + 5x - 10 = A(x-1)(x+2) + B(x+2) + C(x-1)$

$x \to 1$ $\quad -3 = 3B \;\Rightarrow\; B = -1$ M1 A1

$x \to -2$ $\quad -12 = -3C \;\Rightarrow\; C = 4$ A1 4

(b) $\dfrac{2x^2+5x-10}{(x-1)(x+2)} = 2 + (1-x)^{-1} + 2\left(1+\dfrac{x}{2}\right)^{-1}$ M1

$(1-x)^{-1} = 1 + x + x^2 + \ldots$ B1

$\left(1+\dfrac{x}{2}\right)^{-1} = 1 - \dfrac{x}{2} + \dfrac{x^2}{4} + \ldots$ B1

$\dfrac{2x^2+5x-10}{(x-1)(x+2)} = (2+1+2) + (1-1)x + \left(1+\dfrac{1}{2}\right)x^2 + \ldots$ M1

$= 5 + \ldots$ ft their $A - B + \frac{1}{2}C$ A1 ft

$= \ldots + \dfrac{3}{2}x^2 + \ldots$ $0x$ stated or implied A1 A1 7

[11]

2. (a) $(1-8x)^{\frac{1}{2}} = 1 + \left(\tfrac{1}{2}\right)(-8x) + \dfrac{\left(\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)}{2}(-8x)^2$

$+ \dfrac{\left(\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)}{\mathbf{3!}}(-8x)^3 + \ldots.$ M1 A1

$= 1 - 4x - 8x^2; -32x^3 - \ldots$ A1 A1 4

(b) $\sqrt{(1-8x)} = \sqrt{\left(1-\dfrac{8}{100}\right)}$ M1

$= \sqrt{\dfrac{92}{100}} = \sqrt{\dfrac{23}{25}} = \dfrac{\sqrt{23}}{5}$ * cso A1 2

(c) $1–4x–8x^2–32x^3=1–4(0.01)–8(0.01)^2 – 32(0.01)^3$

$=1– 0.04– 0.0008–0.000\ 032 = 0.959168$ M1

$\sqrt{23} = 5\times 0.959168$ M1

$= 4.795\ 84$ cao A1 3

[9]

3. $\text{f}(x) = \dfrac{1}{\sqrt{(4+x)}} = (4+x)^{-\frac{1}{2}}$ M1

$= (4)^{-\frac{1}{2}}(1+....)^{...}$ $\quad \dfrac{1}{2}(1+....)^{...}$ or $\dfrac{1}{2\sqrt{(1+...)}}$ B1

$= ...\left(1 + (-\tfrac{1}{2})\left(\dfrac{x}{4}\right) + \dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2}\left(\dfrac{x}{4}\right)^2 + \dfrac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}\left(\dfrac{x}{4}\right)^3 + ...\right)$ M1 A1ft

ft their $\left(\dfrac{x}{4}\right)$

$= \dfrac{1}{2} - \dfrac{1}{16}x, + \dfrac{3}{256}x^2 - \dfrac{5}{2048}x^3 + ...$ A1, A1 6

Alternative

$\text{f}(x) = \dfrac{1}{\sqrt{(4+x)}} = (4+x)^{-\frac{1}{2}}$ M1

$= \underline{4^{-\frac{1}{2}}} + (-\tfrac{1}{2})4^{-\frac{3}{2}}x + \dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2}4^{-\frac{5}{2}}x^2 + \dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{1.2.3}4^{-\frac{7}{2}}x^3 + ...$ <u>B1</u> M1 A1

$= \dfrac{1}{2} - \dfrac{1}{16}x, + \dfrac{3}{256}x^2 - \dfrac{5}{2048}x^3 + ...$ A1, A1 6

[6]

4. (a) $27x^2 + 32x + 16 \equiv A(3x + 2)(1 - x) + B(1 - x) + C(3x + 2)^2$ Forming this identity Substitutes either $x = -\frac{2}{3}$ or M1

$x = -\frac{2}{3}, 12 - \frac{64}{3} + 16 = \left(\frac{5}{3}\right)B \Rightarrow \frac{20}{3} = \left(\frac{5}{3}\right)B \Rightarrow B = 4$ $x = 1$ into their identity or equates 3 terms or substitutes in values to write down three M1

$x = 1, 27 + 32 + 16 = 25C \Rightarrow 75 = 25C \Rightarrow C = 3$ simultaneous equations. Both $B = 4$ and $C = 3$ A1

(**Note the A1 is dependent on both method marks in this part.)**

Equate x^2: $27 = -3A + 9C \Rightarrow 27 = -3A + 27 \Rightarrow 0 = -3A$

$\Rightarrow A = 0$ Compares coefficients or substitutes in a third x-value or uses simultaneous

$x = 0, 16 = 2A + B + 4C$ equations to show $A = 0$. B1 4

$\Rightarrow 16 = 2A + 4 + 12 \Rightarrow 0 = 2A \Rightarrow A = 0$

(b) $f(x) = \dfrac{4}{(3x+2)^2} + \dfrac{3}{(1-x)}$

$= 4(3x + 2)^{-2} + 3(1 - x)^{-1}$ Moving powers to top on any one of the two expressions M1

$= 4\left[2(1 + \frac{3}{2}x)^{-2}\right] + 3(1 - x)^{-1}$

$= \left(1 + \frac{3}{2}x\right)^{-2} + 3(1 - x)^{-1}$

$= 1\left\{\underline{1 + (-2)(\tfrac{3x}{2}); + \dfrac{(-2)(-3)}{2!}(\tfrac{3x}{2})^2 + \ldots}\right\}$ Either $1 \pm (-2)(\frac{3x}{2})$; or $1 \pm (-1)(-x)$ from either first or second expansions respectively

Ignoring 1 and 3, any one

$+ 3\left\{\underline{1 + (-1)(-x); + \dfrac{(-1)(-2)}{2!}(-x)^2 + \ldots}\right\}$ correct {<u>...........</u>} A1

expansion. Both {<u>...........</u>} correct. A1

$= \left\{1 - 3x + \frac{27}{4}x^2 + \ldots\right\} + 3\left\{1 + x + x^2 + \ldots\right\}$

$= 4 + 0x; + \frac{39}{4}x^2$ $4 + (0x)$; $\frac{39}{4}x^2$ A1; A1 6

(c) Actual = f (0.2) = $\frac{1.08+6.4+16}{(6.76)(0.8)}$ Attempt to find the actual value of f(0.2)

= $\frac{23.48}{5.408} = 4.341715976... = \frac{2935}{676}$ or seeing awrt 4.3 and believing it is candidate's actual f(0.2). M1

Or

Actual = f (0.2) = $\frac{4}{(3(0.2)+2)^2} + \frac{3}{(1-0.2)}$

Candidates can also attempt to find the actual value by using $\frac{A}{(3x+2)} + \frac{B}{(3x+2)^2} + \frac{C}{(1-x)}$

= $\frac{4}{6.76} + 3.75 = 4.341715976... = \frac{2935}{676}$ with their A, B and C.

Estimate = f(0.2) = $4 + \frac{39}{4}(0.2)^2$ Attempt to find an estimate for f(0.2) using their answer to (b) M1ft

= 4 + 0.39 = 4.39

%age error = $\frac{|4.39 - 4.341715976...|}{4.341715976...} \times 100$

$\left|\frac{\text{their estimate - actual}}{\text{actual}}\right| \times 100$ M1

= 1.112095408... = 1.1%(2sf) 1.1% A1 cao 4

[14]

5. ** represents a constant (which must be consistent for first accuracy mark)

(a) $\frac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}} = \underline{(4)^{-\frac{1}{2}}}\left(1 - \frac{3x}{4}\right)^{-\frac{1}{2}} = \underline{\frac{1}{2}}\left(1 - \frac{3x}{4}\right)^{-\frac{1}{2}}$

$-\frac{1}{2}\left[\underline{1 + \left(-\frac{1}{2}\right)(**x); + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(**x)^2 + ...}\right]$

with ** ≠ 1

$= \frac{1}{2}\left[\underline{1 + \left(-\frac{1}{2}\right)\left(-\frac{3x}{4}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(-\frac{3x}{4}\right)^2 + ...}\right]$

$= \frac{1}{2}\left[1 + \frac{3}{8}x; + \frac{27}{128}x^2 + ...\right]$

$\left\{= \frac{1}{2} + \frac{3}{16}x; + \frac{27}{256}x^2 + \ldots\right\}$ *Ignore subsequent working*

$\underline{(4)^{-\frac{1}{2}}}$ or $\frac{1}{\underline{2}}$ outside brackets B1

Expands $(1 + ** x)^{-\frac{1}{2}}$ to give a simplified or an un-simplified

$1 + \left(-\frac{1}{2}\right)(** x)$; M1

A correct simplified or an un-simplified [<u>..........</u>] expansion with candidate's followed through ($**x$) A1ft

Award SC M1 if you see $\left(-\frac{1}{2}\right)(** x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(** x)^2$

$\frac{1}{2}\left[1 + \frac{3}{8}x; \ldots\right]$

SC: $K\left[1 + \frac{3}{8}x + \frac{27}{128}x^2 + \ldots\right]$ A1 isw

$\frac{1}{2}\left[\ldots\ldots\ldots\ldots; \frac{27}{128}x^2\right]$ A1 isw 5

(b) $(x + 8)\left(\frac{1}{2} + \frac{3}{16}x + \frac{27}{256}x^2 + \ldots\right)$

$= \quad \frac{1}{2}x + \frac{3}{16}x^2 + \ldots.$

$\quad + 4 + \frac{3}{2}x + \frac{27}{32}x^2 + \ldots$

$= 4 + 2x; + \frac{33}{32}x^2 + \ldots$

Writing (x + 8) multiplied by candidate's part (a) expansion. M1

Multiply out brackets to find a constant term, two x terms and two x^2 terms. M1

Anything that cancels to 4 + 2x; $\frac{33}{32}x^2$ A1; A1 4

[9]

6. ** represents a constant (which must be consistent for first accuracy mark)

(a) $(8-3x)^{\frac{1}{3}} = \underline{(8)^{\frac{1}{3}}}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}}$

$$= 2\left\{\underline{1+\left(\frac{1}{3}\right)(**x);+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}(**x)^2+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}(**x)^3+\ldots}\right\}$$

with ** ≠ 1

Award SC M1 if you see $\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}(**x)^2+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}(**x)^3$

$$= 2\left\{\underline{1+\left(\frac{1}{3}\right)\left(-\frac{3x}{8}\right);+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(-\frac{3x}{8}\right)^2+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}\left(-\frac{3x}{8}\right)^3+\ldots}\right\}$$

$$= 2\left\{1-\frac{1}{8}x;-\frac{1}{64}x^2-\frac{5}{1536}x^3-\ldots\right\}$$

$$= 2-\frac{1}{4}x;-\frac{1}{32}x^2-\frac{5}{768}x^3-\ldots$$

Takes 8 outside the bracket to give any of $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$. B1

Expands $(1+**x)^{\frac{1}{3}}$ to give a simplified or an un-simplified $1+\left(\frac{1}{3}\right)(**x)$; M1;

A correct simplified or an un-simplified {<u>............</u>} expansion with candidate's followed through (**x) A1ft

Either $2\left\{1-\frac{1}{8}x\ldots\ldots\ldots\ldots\right\}$ or anything that cancels to $2-\frac{1}{4}x$; A1; 5

(b) $(7.7)^{\frac{1}{3}} \approx 2-\frac{1}{4}(0.1)-\frac{1}{32}(0.1)^2-\frac{5}{768}(0.1)^3-\ldots$

= 2 – 0.025 – 0.0003125 – 0.0000065104166...

= 1.97468099...

Attempt to substitute x = 0.1 into a candidate's binomial expansion. M1

awrt 1.9746810 A1 2

You would award B1M1A0 for

$$= 2\left\{ !+\left(\frac{1}{3}\right)\left(-\frac{3x}{8}\right)+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(-\frac{3x}{8}\right)^2+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}(-3x)^3+\ldots \right\}$$

because ** is not consistent.

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822...$

Aliter

(a) **Way 2**

$(8-3x)^{\frac{1}{3}}$

$$= \left\{ (8)^{\frac{1}{3}} + \left(\frac{1}{3}\right)(8)^{-\frac{2}{3}}(**x); + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}(8)^{-\frac{5}{3}}(**x)^2 + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}(8)^{-\frac{8}{3}}(**x)^3 + \ldots \right\}$$

with $** \neq 1$

$$= \left\{ (8)^{\frac{1}{3}} + \left(\frac{1}{3}\right)(8)^{-\frac{2}{3}}(-3x); + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}(8)^{-\frac{5}{3}}(-3x)^2 + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}(8)^{-\frac{8}{3}}(-3x)^3 + \ldots \right\}$$

Award SC M1 if you see $\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}(8)^{-\frac{5}{3}}(**x)^2 + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}(8)^{-\frac{8}{3}}(**x)^3$

$$= \left\{ 2 + \left(\frac{1}{3}\right)\left(\frac{1}{4}\right)(-3x) + \left(-\frac{1}{9}\right)\left(\frac{1}{32}\right)(9x^2) + \left(\frac{5}{81}\right)\left(\frac{1}{256}\right)(-27x^3) + \ldots \right\}$$

$$= 2 - \frac{1}{4}x; - \frac{1}{32}x^2 - \frac{5}{768}x^3 - \ldots$$

2 or $(8)^{\frac{1}{3}}$ (**See note ↓**) B1

Expands $(8-3x)^{\frac{1}{3}}$ to give an un-simplified or simplified

$(8)^{\frac{1}{3}} + \left(\frac{1}{3}\right)(8)^{-\frac{2}{3}}(**x)$; M1;

A correct un-simplified or simplified {.........} expansion with candidate's followed through (** x) A1ft

Anything that cancels to $2-\frac{1}{4}x$; A1;

or $2\left\{1-\frac{1}{8}x..........\right\}$

Simplified $-\frac{1}{32}x^2-\frac{5}{768}x^3$ A1 5

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822...$

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

[7]

7. $f(x) = (3 + 2x)^{-3} = \underline{(3)^{-3}}\left(1+\frac{2x}{3}\right)^{-3} = \underline{\frac{1}{27}}\left(1+\frac{2x}{3}\right)^{-3}$

$$= \frac{1}{27}\left\{\underline{1+(-3)(**x);\frac{(-3)(-4)}{2!}(**x)^2+\frac{(-3)(-4)(-5)}{3!}(**x)^3+...}\right\}$$

with ** ≠ 1 5

B1 Takes 3 outside the bracket to give any of $(3)^{-3}$ or $\frac{1}{27}$.

See note below.

M1; Expands $(1 + **x)^{-3}$ to give a simplified or an un-simplified $1 + (-3)(**x)$;

A1ft A correct simplified or an un-simplified{..........} expansion with candidate's followed thro' (**x)

$$= \frac{1}{27}\left\{\underline{1+(-3)(\frac{2x}{3});\frac{(-3)(-4)}{2!}(\frac{2x}{3})^2+\frac{(-3)(-4)(-5)}{3!}(\frac{2x}{3})^3+...}\right\}$$

$$= \frac{1}{27}\left\{1-2x+\frac{8x^2}{3}-\frac{80}{27}x^3+...\right\}$$

$$= \frac{1}{27}-\frac{2x}{27};+\frac{8x^2}{81}-\frac{80x^3}{729}+...$$

A1; Anything that cancels to $\frac{1}{27}-\frac{2x}{27}$;

A1 Simplified $\frac{8x^2}{81}-\frac{80x^3}{729}$

Way 2

$f(x) = (3 + 2x)^{-3}$

$$= \left\{ \begin{array}{l} (3)^{-3} + (-3)(3)^{-4}(**x); + \frac{(-3)(-4)}{2!}(3)^{-5}(**x)^2 \\ \qquad + \frac{(-3)(-4)(-5)}{3!}(3)^{-6}(**x)^3 + \ldots \end{array} \right\}$$

with $** \neq 1$

B1 $\frac{1}{27}$ or $(3)^{-3}$ (**See note below**)

M1 Expands $(3 + 2x)^{-3}$ to give and un-simplified $(3)^{-3} + (-3)(3)^{-4}(**x)$;

A1ft A correct un-simplified or simplified {..........} expansion with candidate's followed thro' $(**x)$

$$= \left\{ \begin{array}{l} (3)^{-3} + (-3)(3)^{-4}(2x); + \frac{(-3)(-4)}{2!}(3)^{-5}(2x)^2 \\ \qquad + \frac{(-3)(-4)(-5)}{3!}(3)^{-6}(2x)^3 + \ldots \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \frac{1}{27} + (-3)\left(\frac{1}{81}\right)(2x); + (6)\left(\frac{1}{243}\right)(4x^2) \\ \qquad + (-10)\left(\frac{1}{729}\right)(8x^3) + \ldots \end{array} \right\}$$

$$= \frac{1}{27} - \frac{2x}{27}; + \frac{8x^2}{81} - \frac{80x^3}{729} + \ldots$$

A1; Anything that cancels to $\frac{1}{27} - \frac{2x}{27}$;

A1 Simplified $\frac{8x^2}{81} - \frac{80x^3}{729}$

Attempts using Maclaurin expansions need to be escalated up to your team leader. If you feel the mark scheme does not apply fairly to a candidate please escalate the response up to your team leader.

Special Case:

If you see the constant $\frac{1}{27}$ in a candidate's final binomial expression, then you can award B1

[5]

8. ** represents a constant

$$f(x) = (2-5x)^{-2} = \underline{(2)^{-2}}\left(1 - \frac{5x}{2}\right)^{-2} = \underline{\frac{1}{4}}\left(1 - \frac{5x}{2}\right)^{-2}$$

$$= \frac{1}{4}\left\{\underline{1 + (-2)(** x); + \frac{(-2)(-3)}{2!}(** x)^2 + \frac{(-2)(-3)(-4)}{3!}(** x)^3 + ...}\right\}$$

Takes 2 outside the bracket to give any of $(2)^{-2}$ or $\frac{1}{4}$ B1

Expands $(1 + ** x)^{-2}$ to give an unsimplified $1 + (-2)(**x)$; M1
A correct unsimplified {.........} expansion with candidate's $(**x)$ A1

$$= \frac{1}{4}\left\{\underline{1 + (-2)\left(\frac{-5x}{2}\right); + \frac{(-2)(-3)}{2!}\left(\frac{-5x}{2}\right)^2 + \frac{(-2)(-3)(-4)}{3!}\left(\frac{-5x}{2}\right)^3 + ...}\right\}$$

$$= \frac{1}{4}\left\{1 + 5x; + \frac{75x^2}{4} + \frac{125x^3}{2} + ...\right\}$$

$$= \frac{1}{4} + \frac{5x}{4}; + \frac{75x^2}{16} + \frac{125x^3}{8} + ...$$

$$= \frac{1}{4} + 1\frac{1}{4}x; + 4\frac{11}{16}x^2 + 15\frac{5}{8}x^3 + ...$$

Anything that cancels to $\frac{1}{4} + \frac{5x}{4}$; A1;

Simplified $\frac{75x^2}{16} + \frac{125x^3}{8}$ A1

[5]

Aliter
Way 2

$f(x) = (2 - 5x)^{-2}$

$$= \left\{\underline{(2)^{-2} + (-2)(2)^{-3}(** x); + \frac{(-2)(-3)}{2!}(2)^{-4}(** x)^2 + \frac{(-2)(-3)(-4)}{3!}(2)^{-5}(** x)^3 + ...}\right\}$$

$\frac{1}{4}$ or $(2)^{-2}$ B1

Expands $(2 - 5x)^{-2}$ to give an unsimplified $(2)^{-2} + (-2)(2)^{-3}(**x)$; M1

A correct unsimplified {.........} expansion with candidate's $(**x)$ A1

$$= \left\{ \begin{array}{c} (2)^{-2} + (-2)(2)^{-3}(-5x); + \frac{(-2)(-3)}{2!}(2)^{-4}(-5x)^2 \\ + \frac{(-2)(-3)(-4)}{3!}(2)^{-5}(-5x)^3 + \ldots \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \frac{1}{4} + (-2)\left(\frac{1}{8}\right)(-5x); + (3)\left(\frac{1}{16}\right)(25x^2) \\ + (-4)\left(\frac{1}{16}\right)(-125x^3) + \ldots \end{array} \right\}$$

$$= \frac{1}{4} + \frac{5x}{4}; + \frac{75x^2}{16} + \frac{125x^3}{8} + \ldots$$

$$= \frac{1}{4} + 1\frac{1}{4}x; + 4\frac{11}{16}x^2 + 15\frac{5}{8}x^3 + \ldots$$

Anything that cancels to $\frac{1}{4} + \frac{5x}{4}$; A1;

Simplified $\frac{75x^2}{16} + \frac{125x^3}{8}$ A1

Attempts using Maclaurin expansions need to be referred to your team leader.

[5]

9. (a) $3x - 1 \equiv A(1 - 2x) + B$

Let $x = \frac{1}{2}$; $\frac{3}{2} - 1 = B$ $\Rightarrow B\ -\frac{1}{2}$ M1

Considers this identity and either substitutes $x = \frac{1}{2}$, equates coefficients or solves simultaneous equations

Equate x terms; $3 = -2A \Rightarrow A = -\frac{3}{2}$ A1;A1 3

$A = -\frac{3}{2}; B = \frac{1}{2}$

***(No working seen,** but A and B correctly stated $\Rightarrow$ award all three marks. If one of A or B correctly stated give two out of the three marks available for this part.)*

(b) $f(x) = -\frac{3}{2}(1-2x)^{-1} + \frac{1}{2}(1-2x)^{-2}$ M1

Moving powers to top on any one of the two expressions

$$= -\tfrac{3}{2}\left\{\underline{1 + (-1)(-2x); + \frac{(-1)(-2)}{2!}(-2x)^2 + \frac{(-1)(-2)(-3)}{3!}(-2x)^3 + \ldots}\right\}$$ dM1

Either 1 ± 2x or 1 ± 4x from either first or second expansions respectively

$$+\tfrac{1}{2}\left\{\underline{1+(-2)(-2x);+\frac{(-2)(-3)}{2!}(-2x)^2+\frac{(-2)(-3)(-4)}{3!}(-2x)^3+\ldots}\right\}$$

Ignoring $-\frac{3}{2}$ *and* $\frac{1}{2}$*,*
any one correct
{……….} expansion. A1
Both {……….} correct. A1

$= \dfrac{3}{2}\{1 + 2x + 4x^2 + 8x^3 + \ldots\} + \dfrac{1}{2}\{1 + 4x + 12x^2 + 32x^3 + \ldots\}$

$= 1 - x\ ; + 0x^2 + 4x^3$ A1; A1 6

$-1\ -x;\ (0x^2) + 4x^3$

[9]

Aliter Way 2

(b) $f(x) = (3x - 1)(1 - 2x)^{-2}$ M1

Moving power to top

$$=(3x-1)\times\left(1+(-2)(-2x);+\frac{(-2)(-3)}{2!}(-2x)^2+\frac{(-2)(-3)(-4)}{3!}(-2x)^3+\ldots\right)$$ dM1;

$1 \pm 4x$;
Ignoring $(3x - 1)$, correct
(…………..) expansion A1

$= (3x - 1)(1 + 4x + 12x^2 + 32x^8 + \ldots)$

$-3x + 12x^2 + 36x^3 - 1 - 4x - 12x^2 - 32x^3 + \ldots$ Correct expansion A1

$= -1 - x\ ; + 0x^2 + 4x^3$ $-1\ -x;\ (0x^2) + 4x^3$ A1; A1 6

Aliter Way 3

(b) Maclaurin expansion

$f(x) = -\tfrac{3}{2}(1-2x)^{-1} + \tfrac{1}{2}(1-2x)^{-2}$ M1

Bringing (b) both powers to top

$f'(x) = -3(1-2x)^{-2} + 2(1-2x)^{-3}$ M1;

Differentiates to give
$a(1-2x)^{-2} \pm b(1-2x)^{-3}$;
$-3(1-2x)^{-2} + 2(1-2x)^{-3}$ A1 oe

$f''(x) = -12(1-2x)^{-3} + 12(1-2x)^{-4}$

$f'''(x) = -72(1-2x)^{-4} + 96(1-2x)^{-5}$ A1

Correct $f''(x)$ and $f'''(x)$

$\therefore f(0) = -1$, $f'(0) = -1$, $f''(0) = 0$ and $f'''(0) = 24$

gives $f(x) = -1 - x; + 0x^2 + 4x^3 + \ldots$ A1; A1 6

$-1 - x$; $(0x^2) + 4x^3$

Aliter Way 4

(b) $f(x) = -3(2-4x)^{-1} + \frac{1}{2}(1-2x)^{-2}$ M1

Moving powers to top on any one of the two expressions

$$= -3\left\{\underline{(2)^{-1} + (-1)(2)^{-2}(-4x); + \frac{(-1)(-2)}{2!}(2)^{-3}(-4x)^2 + \frac{(-1)(-2)(-3)}{3!}(2)^{-4}(-4x)^3 + \ldots}\right\}$$ dM1;

Either $\frac{1}{2} \pm x$ or $1 \pm 4x$ from either first or second expansions respectively

$$+ \tfrac{1}{2}\left\{\underline{1 + (-2)(-2x); + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \ldots}\right\}$$

Ignoring -3 and $\frac{1}{2}$,
any one correct
{…………} expansion. A1
Both {………..} correct. A1

$$= -3\left\{\tfrac{1}{2} + x + 2x^2 + 4x + \ldots\right\} + \tfrac{1}{2}\left\{1 + 4x + 12x^2 + 32x^3 + \ldots\right\}$$

$= 1 - x; +0x^2 + 4x^3$ $-1 - x$; $(0x^2) + 4x^3$ A1; A1 6

10. (a) $(1+\left(-\frac{1}{2}\right)(-2x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{1.2}(-2x)^2+\frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{1.2.3}(-2x)^3+....)$

M1 (corr bin coeffs)
M1 (powers of $-2x$)

$= 1 + x, + \frac{3}{2}x^2 + \frac{5}{2}x^3 + ...$ A1 A1 4

Alternative

May use McLaurin f(0) = 1 and f′(0) = 1 to obtain 1st two terms $1 + x$ M1 A1
Differentiates two further times and uses formula with correct factorials to give M1

$\frac{3}{2}x^2 + \frac{5}{2}x^3$ A1 4

(b) $(100-200x)^{-\frac{1}{2}} = 100^{-\frac{1}{2}}(1-2x)^{-\frac{1}{2}}.$

So series is $\frac{1}{10}$ (previous series) M1A1 ft 2

[6]

11. (a) Considers $3x^2 + 16 = A(2 + x)^2 + B(1 - 3x)(2 + x) + C(1 - 3x)$ M1

and substitutes $x = -2$, or $x = 1/3$,

or compares coefficients and solves simultaneous equations

To obtain A = 3, and C = 4 A1 A1

Compares coefficients or uses simultaneous equation to show B = 0. B1 4

(b) Writes $3(1 - 3x)^{-1} + 4(2 + x)^{-2}$ M1

$= 3(1 + 3x, + 9x^2 + 27x^3 + ...)+$ (M1 A1)

$\frac{4}{4}(1+\frac{(-2)}{1}\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{1.2}\left(\frac{x}{2}\right)^2+\frac{(-2)(-3)(-4)}{1.2.3}\left(\frac{x}{2}\right)^3+....)$ (M1 A1)

$= 4 + 8x, \quad + 27\tfrac{3}{4}x^2 + 80\tfrac{1}{2}x^3 + ...$ A1 A1

Or uses $(3x^2 + 16)(1 - 3x)^{-1}(2 + x)^{-2}$ M1

$(3x^2 + 16)\ (1 + 3x, + 9x^2 + 27x^3 +) \times$ (M1A1)×

$\frac{1}{4}\left(1 + \frac{(-2)}{1}\left(\frac{x}{2}\right) + \frac{(-2)(-3)}{1.2}\left(\frac{x}{2}\right)^2 + \frac{(-2)(-3)(-4)}{1.2.3}\left(\frac{x}{2}\right)^3\right)$ A1, A1

$= 4 + 8x, \quad + 27\tfrac{3}{4}x^2 + 80\tfrac{1}{2}\ x^3 + \ldots$ 7

[11]

12. $(4 - 9x)^{\frac{1}{2}} = 2\left(1 - \frac{9x}{4}\right)^{\frac{1}{2}}$ B1

$= 2\left(1 + \frac{\frac{1}{2}}{1}\left(-\frac{9x}{4}\right) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1.2}\left(-\frac{9x}{4}\right)^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2.3}\left(-\frac{9x}{4}\right)^3 + \ldots\right)$ M1

$= 2\left(1 - \frac{9}{8}x - \frac{81}{128}x^2 - \frac{729}{1024}x^3 + \ldots\right)$

$= 2 - \frac{9}{4}x, - \frac{81}{64}x^2, - \frac{729}{512}x^3 + \ldots$ A1, A1, A1 5

Note: The M1 is gained for $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1.2}(\ldots)^2$ *or* $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2.3}(\ldots)^3$

Special case

If the candidate reaches $= 2\left(1 - \frac{9}{8}x - \frac{81}{128}x^2 - \frac{729}{1024}x^3 + \ldots\right)$

and goes no further allow A1 A0 A0

[5]

13. (a) $(1 - x)^{-\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(-x)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}(-x)^3$ M1A1

$\left(= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^2 \ldots\right)$

$(1 - x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}x^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}x^3$ M1A1

$\left(= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 \ldots\right)$

$\therefore f(x) = \underline{\frac{1}{2}x^2 + \frac{1}{4}x^3}$ M1A1 6

c.a.o.

	(b)	$f'(x) = x + \frac{3}{4}x^2 \ldots$, $f''(x) = 1 + \frac{3}{2}x\ldots$	M1M1	
		$f(0) = 0$ and $f'(0) = 0$	B1	
		$f''(0) > 0$, $\therefore$ minimum at origin (*)	A1c.s.o.	4

[10]

14.	(a)	$3 + 5x \equiv A(1 - x) + B(1 + 3x)$ Method for A or B	M1	
		$(x = 1) \Rightarrow 8 = 4B$ $B = 2$	A1	
		$(x = -\frac{1}{3}) \Rightarrow \frac{4}{3} = \frac{4}{3}A$ $A = 1$	A1	3
	(b)	$2(1 - x)^{-1} = 2[1 + x + x^2 + \ldots]$	M1 [A1]	
		Use of binomial with n = –1 scores M1(×2)		
		$(1 + 3x)^{-1} = [1 - 3x + \frac{(-1)(-2)}{2!}(3x)^2 + \ldots]$	M1 [A1]	
		$\therefore \frac{3+5x}{(1-x)(1+3x)} = 2 + 2x + 2x^2 + 1 - 3x + 9x^2 = \underline{3 - x + 11x^2}$	A1	5
	(c)	$(1 + 3x)^{-1}$ requires $\lvert x\rvert < \frac{1}{3}$, so expansion is *not* valid.	M1, A1	2

[10]

15.	(a)	Method using either	M1	
		$\frac{A}{(1-x)} + \frac{B}{(2x+3)} + \frac{C}{(2x+3)^2}$ **or** $\frac{A}{1-x} + \frac{Dx+E}{(2x+3)^2}$		
		A = 1	B1,	
		C = 10 , B = 2 **or** D = 4 and E = 16	A1, A1	4
	(b)	$\int [\frac{1}{1-x} + \frac{2}{2x+3} + 10(2x+3)^{-2}]dx$ **or** $\int \frac{A}{1-x} + \frac{Dx+E}{(2x+3)^2}dx$	M1	
		$-\ln\lvert 1 - x\rvert + \ln\lvert 2x + 3\rvert - 5(2x + 3)^{-1}$ $(+c)$ **or**		
		$-\ln\lvert 1 - x\rvert + \ln\lvert 2x + 3\rvert - (2x + 8)(2x + 3)^{-1}$ $(+c)$	M1 A1 ftA1 ftA1 ft	5

	(c)	Either		
		$(1-x)^{-1} + 2(3+2x)^{-1} + 10(3+2x)^{-2} =$	M1	
		$1 + x + x^2 + \ldots$	A1 ft	
		$+\frac{2}{3}(1-\frac{2x}{3}+\frac{4x^2}{9}...)$	M1 A1 ft	
		$+\frac{10}{9}(1+(-2)(\frac{2x}{3})+\frac{(-2)(-3)}{2}(\frac{2x}{3})^2+...)$	A1 ft	
		$=\frac{25}{9}-\frac{25}{27}x+\frac{25}{9}x^2...$	M1 A1	7
		Or		
		$25[(9+12x+4x^2)(1-x)]^{-1} = 25[(9+3x-8x^2-4x^3)]^{-1}$	M1 A1	
		$\frac{25}{9}\left[1+\frac{3x}{9}-\frac{8x^2}{9}-\frac{4x^3}{9}\right]^{-1} = \frac{25}{9}\left[1-\left(\frac{3x}{9}-\frac{8x^2}{9}-\frac{4x^3}{9}\right)+\left(\frac{x^2}{9}..\right).\right]$	M1 A1 A1	
		$=\frac{25}{9}-\frac{25}{27}x+\frac{25}{9}x^2$	M1, A1	7

[16]

16.	(a)	$(1+3x)^{-2} = 1 + (-2)(3x) + \frac{(-2)(-3)}{2!}(3x)^2 + \frac{(-2)(-3)(-4)}{3!}(3x)^3 + \ldots$	M1	
		$= 1, -6x, +27x^2 ...(-108x^3)$	B1, A1, A1	4
	(b)	Using (a) to expand $(x+4)(1+3x)^{-2}$or complete method to find coefficients		
		[e.g. Maclaurin or $\frac{1}{3}(x+3x)^{-1} + \frac{11}{3}(1+3x)^{-2}$].		
		$= 4 - 23x, + 102x^2, -405x^3 = 4, -23x, +102x^2...(-405x^3)$	A1, A1 ft, A1 ft	4

[8]

17	(a)	$1+\frac{3}{4}(12x)+\frac{\frac{3}{4}\left(-\frac{1}{4}\right)}{1\times 2}(12x)^2+\frac{\frac{3}{4}\left(-\frac{1}{4}\right)\left(-\frac{5}{4}\right)}{1\times 2\times 3}(12x)^3$	M1 A1	
		$= 1+9x \quad -\frac{27}{2}x^2 \quad +\frac{135}{2}x^3$		
		$p=-\frac{27}{2}, \quad q=\frac{135}{2}$	A1 A1	4

	(b)	$(1+12x)=1.6 \quad \Rightarrow 12x = 0.6 \quad \Rightarrow x = 0.05$	M1	
		$(1+12x)^{\frac{3}{4}} \quad = 1 + 0.45 - 0.03375 + 0.0084375$ $= 1.4246875$	A1	2
	(c)	From calculator 1.4226235	B1	
		Only accurate to 3 significant figures 1.42	B1	2

[8]

18.	(a)	$\dfrac{1+14x}{(1-x)(1+2x)} \equiv \dfrac{A}{1-x} + \dfrac{B}{1+2x}$ and attempt A and or B	M1	
		$A = 5$, $B = -4$	A1, A1	3
	(b)	$\displaystyle\int \frac{5}{1-x} - \frac{4}{1+2x}\,\mathrm{d}x = [-5\ln\|1-x\| - 2\ln\|1+2x\|]$	M1 A1	
		$= (-5\ln\frac{2}{3} - 2\ln\frac{5}{3}) - (-5\ln\frac{5}{6} - 2\ln\frac{4}{3})$	M1	
		$= 5\ln\frac{5}{4} + 2\ln\frac{4}{5}$		
		$= 3\ln\frac{5}{4} = \ln\frac{125}{64}$	M1 A1	5
	(c)	$5(1-x)^{-1} - 4(1+2x)^{-1}$	B1 ft	
		$= 5(1 + x + x^2 + x^3) - 4$ $\left(1 - 2x + \dfrac{(-1)(-2)(2x)^2}{2} + \dfrac{(-1)(-2)(-3)(2x)^3}{6} + \ldots\right)$	M1 A1	
		$= 1 + 13x - 11x^2 + 37x^3 \ldots.$	M1 A1	5

[13]

1. The first part of question 5 was generally well done. Those who had difficulty generally tried to solve sets of relatively complicated simultaneous equations or did long division obtaining an incorrect remainder. A few candidates found B and C correctly but either overlooked finding A or did not know how to find it. Part (b) proved very testing. Nearly all were able to make the connection between the parts but there were many errors in expanding both $(x - 1)^{-1}$ and $(2 + x)^{-1}$. Few were able to write $(x - 1)^{-1}$ as $-(1 - x)^{-1}$ and the resulting expansions were incorrect in the majority of cases, both $1 + x - x^2$ and $1 - x - x^2$ being common.

$(2 + x)^{-1}$ was handled better but the constant $\frac{1}{2}$ in $\frac{1}{2}\left(1+\frac{x}{2}\right)^{-1}$ was frequently incorrect. Most recognised that they should collect together the terms of the two expansions but a few omitted their value of A when collecting the terms.

2. This proved a suitable starting question and there were many completely correct solutions. The majority of candidates could complete part (a) successfully. In part (b), those who realised that working in common (vulgar) fractions was needed usually gained the method mark but, as noted in the introduction, the working needed to establish the printed result was frequently incomplete. It is insufficient to write down $\sqrt{1-\frac{8}{100}} = \frac{\sqrt{23}}{5}$. The examiners accepted, for example, $\sqrt{1-\frac{8}{100}} = \sqrt{\frac{92}{100}} = \sqrt{\frac{23}{25}} = \frac{\sqrt{23}}{5}$. In part (c), most candidates realised that they had to evaluate their answer to part (a) with $x = 0.01$. However many failed to recognise the implication of part (b), that this evaluation needed to be multiplied by 5. It was not uncommon for candidates to confuse parts (b) and (c) with the expansion and decimal calculation appearing in (b) and fraction work leading to $\sqrt{23}$ appearing in (c).

3. This proved a suitable starting question and the majority of candidates gained 5 or 6 of the available 6 marks. Nearly all could obtain the index as $-\frac{1}{2}$ but there were a minority of candidates who had difficulty in factorising out 4 from the brackets and obtaining the correct multiplying constant of $\frac{1}{2}$. Candidates' knowledge of the binomial expansion itself was good and, even if they had an incorrect index, they could gain the method mark here. An unexpected number of candidates seemed to lose the thread of the question and, having earlier obtained the correct multiplying factor $\frac{1}{2}$ and expanded $\left(1+\frac{x}{4}\right)^{-\frac{1}{2}}$ correctly, forgot to multiply their expansion by $\frac{1}{2}$.

4. Part (a) was tackled well by many candidates. The majority of candidates were able to write down the correct identity. The most popular strategy at this stage (and the best!) was for candidates to substitute $x = 1$ and $x = -\frac{2}{3}$ into their identity to find the values of the constants B and C. The substitution of $x = -\frac{2}{3}$ caused problems for a few candidates which led them to find an incorrect value for B. Many candidates demonstrated that constant A was zero by use of a further value of x or by comparing coefficients in their identity. A significant minority of candidates manipulated their original identity and then compared coefficients to produce three equations in order to solve them simultaneously.

In part (b), most candidates were able to rewrite their partial fractions with negative powers and apply the two binomial expansions correctly, usually leading to the correct answer. A significant minority of candidates found the process of manipulating $4(3x + 2)^{-2}$ to $(1 + \frac{3}{2}x)^{-2}$ challenging.

A significant number of candidates were unsure of what to do in part (c). Some candidates found the actual value only. Other candidates found the estimated value only. Of those who progressed further, the most common error was to find the difference between these values and then divide by their estimate rather than the actual value. Some candidates did not follow the instruction to give their final answer correct to 2 significant figures and thus lost the final accuracy mark.

5. This question was also generally well tackled with about 50% of candidates obtaining at least 8 of the 9 marks available. A substantial minority of candidates were unable to carry out the first step of writing $\frac{1}{\sqrt{(4-3x)}}$ as $\frac{1}{2}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$, with the $\frac{1}{2}$ outside the brackets usually written incorrectly as either 2 or 4. Many candidates were able to use a correct method for expanding a binomial expression of the form $(1 + ax)^n$. A variety of incorrect values of a and n, however, were seen by examiners with the most common being a as $\frac{3}{4}$, 3 and –3 and n as $\frac{1}{2}$, –1 and –2. Some candidates, having correctly expanded $\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$, forgot to multiply their expansion by $\frac{1}{2}$. As expected, sign errors, bracketing errors and simplification errors were also seen in this part. A significant minority of candidates expanded as far as x^3, and were not penalised on this occasion.

In part (b), most candidates realised that they needed to multiply $(x + 8)$ by their expansion from part (a) although a small minority attempted to divide $(x + 8)$ by their expansion. A surprising number of candidates attempted to expand $(x + 8)$ to obtain a power series. Other candidates omitted the brackets around x+ 8 although they progressed as if “invisible” brackets were there. The mark scheme allowed candidates to score 2 marks out of 4 even if their answer in (a) was incorrect and many candidates were able to achieve this.

6. In part (a), a majority of candidates produced correct solutions, but a minority of candidates were unable to carry out the first step of writing $(8-3x)^{\frac{1}{3}}$ as $2\left(1-\frac{3x}{8}\right)^{\frac{1}{3}}$. Those who did so were able to complete the remainder of this part but some bracketing errors, sign errors and manipulation errors were seen.

In part (b), many candidates realised that they were required to substitute $x = 0.1$into their binomial expansion. About half of the candidates were able to offer the correct answer to 7 decimal places, but some candidates made calculation errors even after finding the correct binomial expansion in part (a). A few candidates used their calculator to evaluate the cube root of 7.7 and received no credit.

7. The majority of candidates produced correct solutions to this question, but a substantial minority of candidates were unable to carry out the first step of writing $(3 + 2x)^{-3}$ as $\frac{1}{27}\left(1+\frac{2x}{3}\right)^{-2}$. Those who were able to do this could usually complete the remainder of the question but some sign errors and manipulation errors were seen. Another common error was for candidates to apply $\frac{n(n-1)}{2!}$ and/or $\frac{n(n-1)(n-2)}{3!}$ in the third and fourth terms of their expansion.

8. The majority of candidates produced correct solutions to this question, but a substantial minority of candidates were unable to carry out the first step of writing $(2 - 5x)$ as $\frac{1}{4}\left(1-\frac{5x}{2}\right)^{-2}$.

Those who were able to do this could usually complete the remainder of the question but some sign errors were seen.

9. In part (a) candidates needed to start with the correct identity; although correct solutions were seen from a good proportion of the candidature, a significant number of candidates started with the wrong identity and thus gained no marks. The most common wrong starting point was to use $3x - \equiv A(1-2x)^2 + B(1-2x)$, but $3x-1\equiv A(1-2x)^2+B$ and $3x-1\equiv A(1-2x)^2+Bx$ also occasionally appeared. Candidates using the first identity often produced answers A= $\frac{1}{2}$, B= $-\frac{3}{2}$; the same values but for the wrong constants. Candidates using the second identity could produce the 'correct' answers B=$\frac{1}{2}$,A=$-\frac{3}{2}$ (eg by setting $x = 0$ and $x = \frac{1}{2}$) but this is fortuitous and clearly gains no marks.
Generally candidates showed a good understanding of the work on expanding series in part (b) and most were able to gain some credit. The mark scheme allowed four marks to be gained for the correct unsimplified expansions, as far as the term in x^3, of $(1-2x)^{-1}$ and $(1-2x)^{-2}$ This helped some candidates who went on to make numerical or sign errors when simplifying their expansions and errors in part (a) only affected the final two accuracy marks.

Candidates who multiplied $(3x–1)$ by the expansion of $(1–2x)^{–2}$ gave solutions that were not dependent on their answers in part (a) and it was not uncommon to see a score of zero marks in part (a) followed by a score of six marks in part (b).

10. Many candidates were successful with part (a). The most frequent error involved the use of powers of $2x$ rather than $(–2x)$. The careless use of brackets led both to sign errors in part (a) and also a significant number of incorrect answers in (b), the most common being either 100, 10 or $\frac{1}{1000}$ × [answer to part (a)].

11. This was a high scoring question for many candidates, with even weaker candidates often picking up half marks.

A problem in part (a) was that some candidates used the fact that $B = 0$ to find A or C or both, especially by candidates who compared coefficients and found themselves getting "bogged down" with manipulating the equations. Candidates who substituted $x = –2$ and $x = \frac{1}{3}$ had an easier route and usually fared better.

The work on the binomial expansions in part (b) was generally well done, with the vast majority of candidates choosing to work with $3(1 – 3x)^{–1} +4(2 + x)^{–2}$ rather than $(3x^2 + 16)(1 –3x)^{–1} (2 + x)^{–2}$; those that did chose the latter route were rarely successful. Whilst dealing with $(2 + x)^{–2}$ did cause problems, the mark scheme did allow candidates to still score highly even if the $2^{–2}$ factor was not correct.

12. In general, this was well done but a substantial minority of candidates were unable to carry out the first step of writing $(4-9x)^{\frac{1}{2}}$ as $2\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$. Those who could do this could usually complete the question but many errors of sign manipulation were seen.

13. The vast majority of candidates were able to demonstrate their understanding of the binomial expansion, scoring highly. The accuracy in evaluating the binomial coefficients was impressive. Miscopying $\sqrt{1+x}$ as $\sqrt{1-x}$ was a common error. In part (b) the majority successfully showed a minimum at the origin, although few confirmed that the curve actually passed through the origin by evaluating $f(0) = 0$.

14. Part (a) was answered very well by almost all the candidates and there were few numerical errors seen. The binomial expansion was used confidently in part (b) but the usual sign slips and lost brackets led to a number of errors. Most used their values of A and B from part (a) and addition to obtain the final expansion but occasionally a candidate tried to multiply the series together. Some weaker candidates thought that $\frac{2}{(1-x)} = (1-x)^{-2}$. Part (c) was not answered well. Some knew the conditions and explained thoroughly that the expansion was only valid for $|x| < \frac{1}{3}$ and therefore $x = \frac{1}{2}$ was not valid. Others tried substituting $x = \frac{1}{2}$ and declared it was invalid when the series gave a different answer to the original expression, and a few thought that the only condition was that $1+3x \neq 0$ and $1-x \neq 0$.

15. Most candidates chose an appropriate form for the partial fractions, and they demonstrated knowledge of how to find values for their constants. Where things did go wrong, it was often because of errors in arithmetic, but also in forming the correct numerator for the combined partial fractions – an extra factor of $2x + 3$ was popular.

Those who chose constant numerators for the fractions had the easier time in part (b), although there were frequent errors involving wrong signs and wrong constants in the integrals. Many candidates could not integrate $(3 + 2x)^{-2}$, and erroneously used a log. Few candidates with the two-fraction option could see how to make progress with the integration of the fraction with the quadratic denominator, though there were a number of possible methods.

In part (c) very few candidates achieved full marks for this part as many made errors with signs when expanding using the binomial. The most common mistake however was to take out the 3 from the brackets incorrectly. Not as $\frac{1}{3}$ and $\frac{1}{9}$, but as 3 and 9. However many candidates did manage to get some marks, for showing they were trying to use negative powers and also for the $1+x+x^2$, and for trying to collect all their terms together at the end.

16. The Binomial expansion caused very few problems and most understood how to use their answer to part (a) to expand the expression given in part (b). An amazing number ignored the constant 4 as the number with the lowest power of x when they came to collect like terms.

17. No Report available for this question.

18. No Report available for this question.